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LETTER TO THE EDITOR

Massless charges without self-interaction

Adam Bednorz

Institute of Theoretical Physics, Warsaw University, 00-681 Warszawa, Hoża 69, Poland

E-mail: abednorz@fuw.edu.pl

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Abstract
The solutions of Maxwell equations with electric and magnetic fields existing only in a finite part of space are given, due to massless charges moving with speed of light. Under proper conditions, there is no interaction between charges and the field.

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1. Introduction

The problem of self-interaction of charges, both classical and quantum, has a very long history [1–5]. The existing methods always start from a massive case with, sometimes, the analysis of the ultrarelativistic (massless) limit. We would like to focus only on the massless case from the very beginning. Although the neglection of mass may seem implausible, we believe that our considerations can be a good approximation for very fast electrons or at a very fundamental level where the mass may arise as a perturbation [6].

Maxwell equations can be solved for an arbitrary distribution of charges and currents satisfying charge conservation law. We know the case without charges and currents—electromagnetic waves, electrostatic and magnetostatic solutions and general Lienard–Wiechert fields [7]. Apart from Lienard–Wiechert, very few exact solutions have been given for currents changing in time-space. A very simple but pretty example is found in the famous Feynmann's lectures [8]. A constant current is induced along an infinite plane creating an electromagnetic wave propagating perpendicularly to the plane.

In this letter, we would like to extend Feynmann's idea to more sophisticated but, we believe, still pretty and instructive cases. The problem is to find such solutions of Maxwell equations that the electromagnetic wave still moves at the speed of light but does not occupy the whole space. Moreover, the finiteness implies existence of charge density moving in the same direction as the fields [9]. The interesting feature of such an object is that no force is exerted by the field on the charges, so the compound is stable. This is true both in classical and in quantum case, but in the case of the quantum Dirac–Weyl equation self-interaction is more difficult to get rid of.

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2. Classical massless charges

We recall Maxwell equations in the form [3]:

$$\begin{aligned} &\nabla\cdot\boldsymbol{B}=0, && \partial_t\boldsymbol{B}=-\nabla\times\boldsymbol{E},\\ &\nabla\cdot\boldsymbol{E}=\rho/\epsilon_0, && \partial_t\boldsymbol{E}+\boldsymbol{j}/\epsilon_0=c^2\nabla\times\boldsymbol{B}, \end{aligned} \tag{1}$$

where $\boldsymbol{E}$, $\boldsymbol{B}$, $\boldsymbol{j}$, ρ, c and ϵ_0 are the electric and magnetic field, current and charge density, the speed of light and electric permittivity, respectively.

The first two equations can be solved automatically by introducing scalar and vector potentials, Φ and $\boldsymbol{A}$, respectively. The fields are given by the relations

$$\boldsymbol{E}=-\nabla\Phi-\partial_t\boldsymbol{A},\qquad \boldsymbol{B}=\nabla\times\boldsymbol{A}. \tag{2}$$

The latter two equations take the form

$$-\Delta\Phi-\partial_t(\nabla\cdot\boldsymbol{A})=\rho/\epsilon_0,\qquad -\partial_t\nabla\Phi-\partial_t^2\boldsymbol{A}+\boldsymbol{j}/\epsilon_0=-\Delta\boldsymbol{A}+\nabla(\nabla\cdot\boldsymbol{A}). \tag{3}$$

Due to gauge invariance, the additional Lorentz gauge equation

$$\partial_t\Phi+c^2\nabla\cdot\boldsymbol{A}=0 \tag{4}$$

is often imposed, because it is consistent with the charge conservation law

$$\partial_t\rho+\nabla\cdot\boldsymbol{j}=0. \tag{5}$$

If equation (4) is satisfied then equations (3) have the form

$$\partial_t^2\Phi-c^2\Delta\Phi=c^2\rho/\epsilon_0\qquad \partial_t^2\boldsymbol{A}-c^2\Delta\boldsymbol{A}=\boldsymbol{j}/\epsilon_0. \tag{6}$$

Usually, one wants to find Φ and $\boldsymbol{A}$ given the charge and current. Moreover, the best known Lienard–Wiechert case—single moving charge—cannot be applied (or is singular at least), when the charge has the speed of light. The textbook solution of the charge with a constant velocity v can be taken with the limit $v\to c$ and the field becomes singular—it forms a flat electromagnetic pulse in the plane passing through the charge and perpendicular to its speed.

We shall concentrate strictly on the solutions at the speed of light in the z-direction, namely

$$\begin{aligned} &\Phi=\Phi(x,y,\xi), && \boldsymbol{A}=\boldsymbol{A}(x,y,\xi),\\ &\rho=\rho(x,y,\xi), && \boldsymbol{j}=(0,0,c\rho), \end{aligned} \tag{7}$$

where $\xi=z-ct$. Taking into account equations (6), we obtain

$$\Delta_2A_x=\Delta_2A_y=\Delta_2(\Phi-cA_z)=0, \tag{8}$$

where $\Delta_2=\partial_x^2+\partial_y^2$. We want our solutions to be finite at infinity so

$$A_x=A_y=0\qquad\text{and}\qquad \Phi=cA_z \tag{9}$$

in the whole time-space. We are left with the equation

$$\Delta_2\Phi=-\rho/\epsilon_0 \tag{10}$$

which is just a two-dimensional electrostatic problem with the parameter ξ. The electric and magnetic fields lie in the plane xy and are given by

$$\boldsymbol{E}=(-\partial_x\Phi,-\partial_y\Phi,0),\qquad \boldsymbol{B}=\hat{\boldsymbol{z}}\times\boldsymbol{E}/c=(\partial_y\Phi/c,-\partial_x\Phi/c,0). \tag{11}$$

Let us calculate the density of Lorentz force

$$\boldsymbol{f}=\rho\boldsymbol{E}+\boldsymbol{j}\times\boldsymbol{B}. \tag{12}$$

Taking into account our solution (11) and equations for current (7) the result is

$$\boldsymbol{f} = 0. \tag{13}$$

The absence of Lorentz force implies that massless charges can propagate without interaction with electromagnetic field on the condition that the movement is in the same direction.

3. Special cases

We shall illustrate our solution (11) by taking several very simple charge distributions in equation (10). Firstly, the Feynmann solution [8]—electromagnetic pulse—is given by

$$\Phi = -Ex\delta(\xi), \qquad \boldsymbol{E} = (E\delta(\xi), 0, 0). \tag{14}$$

Secondly, the well-known moving point charge is represented by

$$\rho = q\delta^2(x, y)\delta(\xi). \tag{15}$$

The solution of (10) is [9, 10]

$$\Phi = -q\ln(r)\delta(\xi)/2\pi\epsilon_0, \qquad \boldsymbol{E} = q\boldsymbol{r}\delta(\xi)/2\pi\epsilon_0 r^2, \tag{16}$$

where $\boldsymbol{r} = (x, y, 0)$ and $r = |\boldsymbol{r}|$.

Both above-mentioned solutions give the electromagnetic field in the whole plane $\xi = 0$. We raise the question: can we have solutions only in a finite area? The following solutions have such a property. Let us take two straight parallel uniformly (and opposite) charged lines, namely

$$\rho = \lambda\delta(\xi)(\delta(x) - \delta(x - a)). \tag{17}$$

The solution is

$$\epsilon_0\Phi = \lambda\delta(\xi)\begin{cases} a & \text{for } x < 0, \\ a - x & \text{for } 0 < x < a, \\ 0 & \text{otherwise} \end{cases} \tag{18}$$

and $\boldsymbol{E} = (E_x, 0, 0)$ with

$$E_x = \begin{cases} \lambda\delta(\xi)/\epsilon_0 & \text{for } 0 < x < a, \\ 0 & \text{otherwise} \end{cases} \tag{19}$$

The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are located on the strip $0 < x < a$ and are absent elsewhere. This solution is finite in x-direction but is still infinite in y-direction.

The final solution is finite in all directions. We put opposite charges q on two coaxial circles with radii a and b, namely

$$\rho = q\delta(\xi)(\delta(r - a)/2\pi a - \delta(r - b)/2\pi b). \tag{20}$$

The electrostatic potential is given by

$$2\pi\epsilon_0\Phi = q\delta(\xi)\begin{cases} -\ln(a/b) & \text{for } r < a, \\ -\ln(r/b) & \text{for } a < r < b, \\ 0 & \text{otherwise} \end{cases} \tag{21}$$

and the field $\boldsymbol{E}$ has the form:

$$\boldsymbol{E} = \begin{cases} q\boldsymbol{r}\delta(\xi)/2\pi\epsilon_0 r^2 & \text{for } a < r < b, \\ 0 & \text{otherwise.} \end{cases} \tag{22}$$

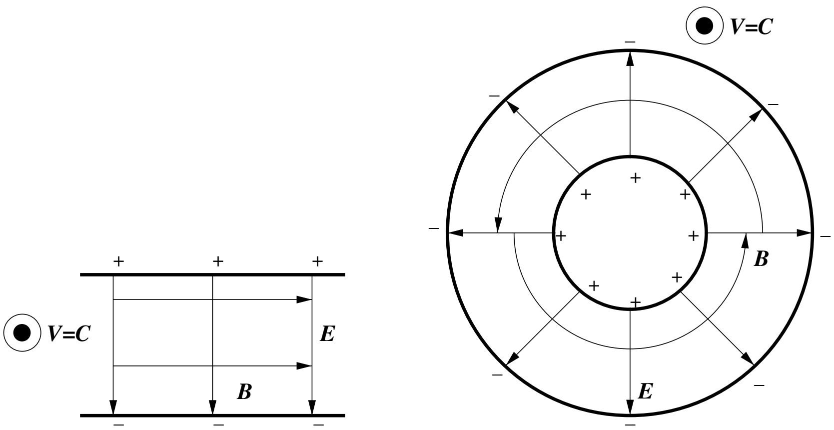

Figure 1. Stable configurations of the electromagnetic field and massless matter. Charges are placed on thick lines and all move up from the page at the speed of light.

The solutions are presented in figure 1. They form a kind of cutting-outs of usual electromagnetic waves. They are stable due to the absence of Lorentz forces.

4. Quantum case

The massless Dirac–Weyl equation for a charge e with the four-component spinor wavefunction ψ in the external field $A^\mu = (\Phi/c, \boldsymbol{A})$ has the form [5]

$$\gamma^\mu(\partial_\mu + \mathrm{i}eA_\mu/\hbar)\psi = 0, \tag{23}$$

where Dirac matrices γ^μ satisfy the relation

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2Ig^{\mu\nu}. \tag{24}$$

We assume the Weyl representation:

$$\gamma^\mu = \begin{bmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{bmatrix} \tag{25}$$

with $\sigma^\mu = (I, \boldsymbol{\sigma})$ and $\bar{\sigma}^\mu = (I, -\boldsymbol{\sigma})$. Pauli matrices are defined as

$$\sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \qquad \sigma^1 = \begin{bmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{bmatrix}, \qquad \sigma^1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}. \tag{26}$$

The spinor ψ can be decoupled into left and right spinors giving two twin Dirac–Weyl equations:

$$\begin{aligned} &(\partial_t + c\boldsymbol{\sigma}\cdot\nabla)\psi_L = \mathrm{i}e(c\boldsymbol{\sigma}\cdot\boldsymbol{A} - \Phi)\psi_L/\hbar, \\ &(\partial_t - c\boldsymbol{\sigma}\cdot\nabla)\psi_R = \mathrm{i}e(-c\boldsymbol{\sigma}\cdot\boldsymbol{A} - \Phi)\psi_R/\hbar, \\ &\psi = \begin{bmatrix} \psi_L \\ \psi_R \end{bmatrix}. \end{aligned} \tag{27}$$

The solution of Dirac equation in an external plane electromagnetic wave was first given by Volkov [11]. Let us consider a very special field from our classical case, namely (9). The spinors of our interest have the form

$$\psi_L = \begin{bmatrix} \psi_{L+}(x, y, \xi) \\ 0 \end{bmatrix}, \qquad \psi_R = \begin{bmatrix} 0 \\ \psi_{R-}(x, y, \xi) \end{bmatrix}. \tag{28}$$

From equations (27), it follows that arbitrary holomorphic functions

$$\psi_{L+}(x + \mathrm{i}y, \xi) \qquad \text{and} \qquad \psi_{R-}(x - \mathrm{i}y, \xi) \tag{29}$$

are solutions of Dirac–Weyl equation.

Note that they receive no corrections from quantum electromagnetic self-interactions because taking charge and current density as

$$\rho = e(|\psi_{L+}|^2 + |\psi_{R-}|^2), \qquad \boldsymbol{j} = (0, 0, c\rho), \tag{30}$$

the self-potentials are given by equations (9) and (10).

The unpleasant feature of our solutions is that they cannot be normalized. Even the trivial case $\psi_{L+} = \psi_{L+}(\xi)$ and $\psi_{R-} = \psi_{R-}(\xi)$ gives the divergent potential:

$$\Phi = x^2\rho/\epsilon_0 \qquad \text{or} \qquad \Phi = r^2\rho/2\epsilon_0. \tag{31}$$

We are unable to repair this defect under Dirac equation but it is possible that quantum field theory as a whole can manage this problem.

5. Discussion

We have shown that the self-interaction vanishes in a very special configuration of charges and electromagnetic fields. It suggests that one can treat masses as a perturbation—a source of self-interaction.

The strange difference between the classical and quantum cases is that not all classical configurations are without self-interaction in quantum description. However, we have only considered a very simple coupling between charges and fields neglecting all difficulties of renormalization, Pauli–Villars masses and other particles. The general quantum field analysis of our problem remains to be conducted.

Acknowledgements

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